A Convergence Result for the Baby Feeding Problem

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***Abstract*— We present new asymptotic convergence results for a variant of the canonical baby feeding problem. Consider a hyperactive squirming baby at feeding time, with a dab of puréed carrots on her chin. The baby proceeds to squirm. At each squirm at $t = 1, \ldots, \infty$, she contacts one area of her body with another. We then pose the question, how many squirms are needed such that carrots cover a 1-ϵ fraction of her body? In this paper we model the baby as a bounded compact set in a Hauserdorff space, and take a Markovian, nonadversarial squirming model. Our analysis proves that convergence is upper bounded by an exponentially decreasing function. Experimental results with a 6 month old infant support the conclusions of our model. We also present extensions of this result to the rate of coverage of the bib, high chair, parents, floors, walls, and ceiling. Future work will consider adversarial squirming, and game-theoretic analysis of the scenario as the parent attempts to clean the baby.**

I. Problem Statement

Let $\mathcal{B}$ represent the surface of the baby. Assume a spherical baby. We assume $\mathcal{B} \subseteq \mathcal{X}$ is a bounded compact submanifold in a measure space $(\mathcal{X}, \mu)$. Without loss of generality, we scale μ appropriately such that $\mu(\mathcal{B}) = 1$. At $t = 0$, a subset $\mathcal{C}$ of $\mathcal{B}$ is covered in carrot purée, or mashed potatoes, or smushed peas, or some other sticky, fluid, nutritious substance which can be spread infinitely thin upon any surface.

Suppose at each time t, the baby squirms. A squirm (f_t, A_t, B_t) is a tuple containing two subsets $A_t, B_t \in \mathcal{B}$, and a bijection $f_t : A_t \mapsto B_t$. The action of the squirm is such that any food at a point $x \in A_t$ is smeared on $f_t(x)$, and any food at a point $y \in B_t$ is smeared on $f_t^{-1}(y)$. Thus, at time t, we can define the sequence $\mathcal{C}_0, \ldots, \mathcal{C}_t$, such that $\mathcal{C}_0 = \mathcal{C}$, and $\mathcal{C}_{t+1}$ is defined by the squirm at t as follows:

$$\mathcal{C}_{t+1} = \{x | x \in \mathcal{C}_t \vee x \in f_t(A_t) \vee x \in f_t^{-1}(B_t). \qquad (1)$$

To squirm, the baby chooses from a set of candidate squirms $\mathcal{S}$. The probability that the baby chooses $S \in \mathcal{S}$ at time t is given by the policy

$$\pi(t, X, S) : \mathbb{R} \times 2^{\mathcal{B}} \times \mathcal{S} \mapsto \mathbb{R} \qquad (2)$$

Let $\alpha_t = \mu(\mathcal{C}_t)$ denote the random variables indicating the fraction of $\mathcal{B}$ covered with food at time t. The baby-feeding problem seeks to ask the following questions:

1) For $x \in \mathcal{B}$, at what t does the probability that $x \in \mathcal{C}_t$ exceed $p > 0$?
2) Does the distribution of α_t converge to 1 as $t \to \infty$?
3) If so, at what time t does $\mu(\mathcal{C}_t)$ exceed $1 - \epsilon$ with probability γ, for constants $\epsilon, \gamma > 0$?

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(a) Before experiment.

(b) After the experiment.

Fig. 1. Subject of experiment, a six month old female infant. Squash purée covered 25.2% of her body and 17.7% of her highchair after three minutes of squirming.

II. A Time-Invariant Markovian Baby

Our analysis addresses the case where $\mu(A_t) = \mu(B_t) = L$, and the choices of squirm $S \in \mathcal{S}$ are identically and independently distributed. We assume that S is chosen uniformly at random. This defines an infinite-state Markov chain on $2^{\mathcal{B}}$, with initial state $\mathcal{C}$ and the transition function is defined such that

$$Pr(x \in \mathcal{C}_{t+1} | x \notin \mathcal{C}_t) = \alpha_k L \qquad (3)$$